

**RIMS**

**Workshop**

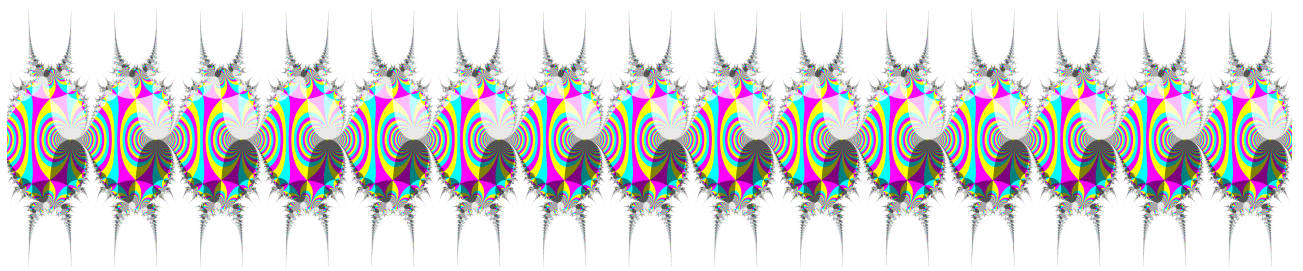
**on**

**Complex Dynamics**

**2017**

RIMS, Kyoto – December 11-15, 2017

[www.math.kyoto-u.ac.jp/~martipete/rimswcd17](http://www.math.kyoto-u.ac.jp/~martipete/rimswcd17)



# PARTICIPANTS

## 参加者

Name	Surname	University	Country	Position
Zin	ARAI	Chubu University	Japan	P
Masayuki	ASAOKA	Kyoto University	Japan	AP
Matthieu	ASTORG	Université d'Orléans	France	AP
Tania Gricel	BENITEZ LÓPEZ	University of Liverpool	UK	D
Fabrizio	BIANCHI	Imperial College London	UK	PD
Romain	DUJARDIN	Université Pierre et Marie Curie	France	P
Vasiliki	EVDORIDOU	IMPAN	Poland	PD
Núria	FAGELLA	Universitat de Barcelona	Spain	AP
Masayo	FUJIMURA	National Defense Academy	Japan	AP
Hiroyuki	INOUE	Kyoto University	Japan	L
Yutaka	ISHII	Kyushu University	Japan	AP
Johannes	JAERISCH	Shimane University	Japan	AP
Atsushi	KAMEYAMA	Gifu University	Japan	P
Tomoki	KAWAHIRA	Tokyo Institute of Technology	Japan	AP
Chunghyun	LEE	Kyushu University	Japan	U
Masashi	KISAKA	Kyoto University	Japan	AP
David	MARTI-PETE	Kyoto University	Japan	PD
Shunsuke	MOROSAWA	Kochi University	Japan	P
Shizuo	NAKANE	Tokyo Polytechnic University	Japan	P
Yusuke	OKUYAMA	Kyoto Institute of Technology	Japan	AP
Leticia	PARDO SIMÓN	University of Liverpool	UK	D
Phil	RIPPON	The Open University	UK	P
Tomoko	SHINOHARA	Tokyo Metropolitan College of Industrial Technology	Japan	AP
Mitsuhiro	SHISHIKURA	Kyoto University	Japan	P
Anand Prakash	SINGH	Central University of Rajasthan	India	P
Gwyneth	STALLARD	The Open University	UK	P
Toshi	SUGIYAMA	Nada High School	Japan	T
Hiroki	SUMI	Kyoto University	Japan	P
Shigeru	TAKEUCHI	Gifu University	Japan	PE
Keisuke	UCHIMURA	Tokai University	Japan	PE
Takato	UEHARA	Saga University	Japan	AP
Kohei	UENO	Daido University	Japan	AP
Shigehiro	USHIKI	Kyoto University	Japan	PE
Tomoo	YOKOYAMA	Kyoto University of Education	Japan	AP
Michel	ZINSMEISTER	Université d'Orléans	France	P

P = Professor    AP = Associate professor    L = Senior lecturer    PE = Professor emeritus

PD = Postdoc    D = PhD student    M = Master student    U = Undergraduate student    T = Teacher

PROGRAM  
プログラム

Monday 11 (月)

- 10:00 - 11:00 **Núria FAGELLA** (Universitat de Barcelona)  
*Wandering domains and singular values*
- 11:20 - 12:20 **Romain DUJARDIN** (Université Pierre et Marie Curie)  
*Degenerations of  $SL(2, C)$  representations, and Lyapunov exponents I*
- 13:50 - 14:50 **Hiroyuki INOU** (Kyoto University)  
*On perturbation of a polynomial with parabolic fixed point*
- 15:10 - 16:10 **Tomoki KAWAHIRA** (Tokyo Institute of Technology)  
*Dynamical and parametric Zalcman functions: Similarity between the Julia sets, the Mandelbrot set, and the tricorn*
- 16:30 - 17:00 **Shunsuke MOROSAWA** (Kochi University)  
*Dynamics of semigroups of transcendental entire functions*

Tuesday 12 (火)

- 9:30 - 10:30 **Shizuo NAKANE** (Tokyo Polytechnic University)  
*On formal normal forms of holomorphic germs at super-saddle fixed points*
- 10:50 - 11:50 **Shigehiro USHIKI** (Kyoto University)  
*Reversible complex dynamical systems and exotic rotation domains*
- 13:20 - 14:20 **Phil RIPPON** (The Open University)  
*The structure of the escaping set of a transcendental entire function*
- 14:40 - 15:40 **Gwyneth STALLARD** (The Open University)  
*Wandering domains and commuting transcendental entire functions*
- 15:55 - 16:25 **Tomoko SHINOHARA** (Tokyo Met. College of Industrial Technology)  
*Local invariant set for a rational map of two variables at a fixed indeterminate point*
- 16:30 - 17:00 **Leticia PARDO-SIMÓN** (University of Liverpool)  
*Escaping singular orbits in the class B*
- 18:00~ **Conference dinner at the restaurant 門 (Mon)**

### Wednesday 13 (水)

- 9:30 - 10:30**      **Matthieu ASTORG** (Université d'Orléans)  
*Collet, Eckmann and the bifurcation measure*
- 10:50 - 11:50**      **Romain DUJARDIN** (Université Pierre et Marie Curie)  
*Degenerations of  $SL(2, C)$  representations, and Lyapunov exponents II*
- 13:20 - 14:20**      **Fabrizio BIANCHI** (Imperial College London)  
*Holomorphic motions of Julia sets*
- 14:40 - 15:40**      **Yusuke OKUYAMA** (Kyoto Institute of Technology)  
*Discontinuity of the escape rate of a degenerating meromorphic family of rational maps*
- 16:00 - 17:00**      **Takato UEHARA** (Saga University)  
*On a construction of transcendental  $K3$  surfaces: application of Arnol'd's theorem*

### Thursday 14 (木)

- 9:30 - 10:30**      **Yutaka ISHII** (Kyushu University)  
 *$M_4$  is regular-closed*
- 10:50 - 11:50**      **Núria FAGELLA** (Universitat de Barcelona)  
*A wandering domain in class  $B$  on which all iterates are univalent*
- 13:20 - 14:20**      **David MARTÍ-PETE** (Kyoto University)  
*Fingers in the parameter space of the complex standard family*
- 14:40 - 15:40**      **Vasiliki EVDORIDOU** (IMPAN)  
*Non-escaping endpoints of disjoint-type functions*
- 15:55 - 16:25**      **Kohei UENO** (Daido University)  
*Bottcher coordinates for holomorphic skew products*
- 16:30 - 17:00**      **Anand Prakash SINGH** (Central University of Rajasthan)  
*On escaping sets of composition of transcendental entire functions*

### Friday 15 (金)

- 9:30 - 10:30**      **Hiroki SUMI** (Kyoto University)  
*Weak mean stability in random holomorphic dynamical systems*
- 10:50 - 11:50**      **Johannes JAERISCH** (Shimane University)  
*Spectral gap property for hyperbolic random complex dynamical systems*
- 12:00 - 13:00**      **Michel ZINSMEISTER** (Université d'Orléans)  
*TBA*

	Mon 11	Tue 12	Wed 13	Thu 14	Fri 15	
		NAKANE	ASTORG	ISHII	SUMI	9:30 - 10:30
10:00 - 11:00	FAGELLA I	Coffee break	Coffee break	Coffee beak	Coffee break	
11:00 - 11:20	Coffee break	USHIKI	DUJARDIN II	FAGELLA II	JAERISCH	10:50 - 11:50
11:20 - 12:20	DUJARDIN I					
12:20 - 13:50	Lunch	Lunch	Lunch	Lunch	ZINSMEISTER	12:00 - 13:00
13:50 - 14:50	INOUE	RIPPON	BIANCHI	MARTÍ-PETE		13:20 - 14:20
14:50 - 15:10	Coffee break	Coffee break	Coffee break	Coffee break		14:20 - 14:40
15:10 - 16:10	KAWAHIRA	STALLARD	OKUYAMA	EVDORIDOU		14:40 - 15:40
16:10 - 16:30	Coffee break	SHINOHARA		Coffee break		15:40 - 15:55
16:30 - 17:00	MOROSAWA	PARDO-SIMÓN	UEHARA	UENO		15:55 - 16:25
				SINGH		16:30 - 17:00

# ABSTRACTS

## アブストラクト

### – COURSES –

**Romain DUJARDIN**, *Degenerations of  $SL(2, \mathbb{C})$  representations, and Lyapunov exponents I and II*

Let  $G$  be a finitely generated group endowed with some probability measure  $\mu$  and  $(\rho_\lambda)$  be a non-compact algebraic family of representations of  $G$  into  $SL(2, \mathbb{C})$ . This can be understood as a random product of Möbius transformations depending on a parameter  $\lambda$ . Using non-Archimedean techniques, we study the asymptotics of this random holomorphic dynamical system as  $\lambda$  goes to infinity. This problem is analogous to that of the description of degenerating families of rational maps by DeMarco-Faber, Favre, and others, and the non-archimedean tools used are similar. This is a report of joint work with Charles Favre.

**Núria FAGELLA**, *Wandering domains and singular values*

TBA.

*A wandering domain in class B on which all iterates are univalent*

TBA.

**Phil RIPPON**, *The structure of the escaping set of a transcendental entire function*

Much recent work on the iterates of a transcendental entire function  $f$  has been motivated by Eremenko's conjecture that all the components of the escaping set  $I(f)$  are unbounded, which is still unsolved. Many partial results on this conjecture are proved using an important subset of  $I(f)$  called the fast escaping set  $A(f)$ . For example,  $A(f)$  can be used to show that  $I(f)$  must contain at least one unbounded component.

Also, there are many examples of entire functions for which  $I(f)$  consists of uncountably many disjoint unbounded curves, most points of which are fast escaping, and also many examples where  $I(f)$  is connected and has the structure of an infinite spider's web. For the set  $A_R(f)$ , which is the 'core' of the fast escaping set, we prove that in a certain sense exactly one of these two structures must happen. This is joint work with Gwyneth Stallard.

**Gwyneth STALLARD**, *Wandering domains and commuting transcendental entire functions*

The classical classification of periodic Fatou components is one of the foundations of complex dynamics. For transcendental functions, Fatou components may be wandering domains and these are much less well understood. For multiply connected wandering domains, however, we do have a complete description of the dynamics within the wandering domain and of the geometry of the wandering domain. We will describe some of the key properties of multiply connected wandering domains from joint work with Walter Bergweiler and Phil Rippon, and more recent work with Phil Rippon on the boundary components of such domains. We also discuss an application of our results in joint work with Phil Rippon and Anna Benini on the question of whether the Julia sets of commuting functions must be equal.

**Michel ZINSMEISTER**, *TBA*

TBA.

**Mathieu ASTORG**, *Collet, Eckmann and the bifurcation measure*

Joint work with T. Gauthier, N. Mihalache and G. Vigny.

The bifurcation measure (in the moduli space of degree  $d$  rational maps) is defined as the Monge-Ampère of the Lyapunov exponent. Its support describes parameters which bifurcate maximally in some sense. We prove that the support of this measure has positive Lebesgue measure in moduli space.

**Fabrizio BIANCHI**, *Holomorphic motions of Julia sets*

For a family of rational maps, results by Lyubich, Mané-Sad-Sullivan and DeMarco provide a fairly complete understanding of dynamical stability. I will review this one-dimensional theory and present a recent generalisation to several complex variables. I will focus on the arguments that do not readily generalise to this setting, and introduce the tools and ideas that allow one to overcome these problems.

**Vasiliki EVDORIDOU**, *Non-escaping endpoints of disjoint-type functions*

Let  $f$  be a transcendental entire function of disjoint type and finite order. The Julia set of  $f$  consists of an uncountable union of disjoint curves each of which joins a finite endpoint to infinity. Following recent results on the topology of the set of non-escaping endpoints for functions in the exponential family, we show that the union of non-escaping endpoints of  $f$  with infinity is a totally separated set.

Combined with a result of Alhabib and Rempe-Gillen this gives a strong dichotomy on the topological properties of the set of endpoints which escape and those which do not escape for disjoint-type functions. This is joint work with D. Sixsmith.

**Hiroyuki INOU**, *On perturbation of a polynomial with parabolic fixed point*

We present a new example of polynomials having a parabolic fixed point, whose perturbation is either in the shift locus or has an attracting fixed point. In particular, such a polynomial cannot be approximated by Misiurewicz polynomials. This is a joint work in progress with Sabyasachi Mukherjee.

**Yutaka ISHII**,  *$\mathcal{M}_4$  is regular-closed*

For each  $n \geq 2$ , we investigate a family of iterated function systems which is parameterized by the unique contraction ratio  $s \in \mathbb{D}^\times \equiv \{s \in \mathbb{C} : 0 < |s| < 1\}$  and possesses a rotational symmetry of order  $n$ . Let  $\mathcal{M}_n$  be the locus of the contraction ratios  $s$  where the corresponding self-similar sets are connected. The purpose of my talk is to show that  $\mathcal{M}_n$  is regular-closed, i.e.  $\text{int } \overline{\mathcal{M}_n} = \mathcal{M}_n$  for  $n \geq 4$ . This gives a new result for  $n = 4$  and a simple geometric proof of the previously known result by Bandt and Hung for  $n \geq 5$ .

**Johannes JAERISCH**, *Spectral gap property for hyperbolic random complex dynamical systems*

We consider hyperbolic random complex dynamical systems on the Riemann sphere with separating condition and multiple minimal sets. We prove the spectral gap property for the transition operators of these systems acting on spaces of Hölder continuous functions. We also investigate the pointwise Hölder exponent of the unitary eigenfunctions of the transition operators.

**Tomoki KAWAHIRA**, *Dynamical and parametric Zalcman functions: Similarity between the Julia sets, the Mandelbrot set, and the tricorn*

We apply Zalcman's lemma to: (1) dynamics of rational maps on the Riemann sphere of degree two or more; and (2) the bifurcation loci of families of rational maps. Then we have families of non-constant meromorphic functions that we call the dynamical and parametric Zalcman functions. In this talk, we present some basic properties of these families following the ideas of Steinmetz. We also give a simple proof of Tan Lei's theorem about the local similarities between the Julia sets and the Mandelbrot set, (and the tricorn) by using the intersection of the families of dynamical and parametric Zalcman functions.

**David MARTÍ-PETE**, *Fingers in the parameter space of the complex standard family*

We study the parameter space of the complex standard family  $F_{\alpha,\beta}(z) = z + \alpha + \beta \sin z$  where the parameter  $0 < \beta \ll 1$  is considered to be fixed and the bifurcation is studied with respect to the parameter  $\alpha \in \mathbb{C}$ . In the real axis of that parameter plane one can observe the so-called Arnold tongues, and from them arise some finger-like structures which were observed for the first time by Fagella in her PhD thesis. Similar structures can also be observed in the parameter spaces of families of Blaschke products or Hénon maps in higher dimension. We study the qualitative and quantitative aspects of the fingers via parabolic bifurcation. This is a work in progress joint with Mitsuhiro Shishikura (Kyoto University).

**Shizuo NAKANE**, *On formal normal forms of holomorphic germs at super-saddle fixed points*

Fiber Julia sets for polynomial skew products is discontinuous if there exists connection between two saddles. In a joint work with Inou, we explained this discontinuity by an analogous argument as parabolic implosion. Here we used linearizing coordinates instead of Fatou coordinates. For this purpose, we had to assume local invertibility at saddles. In case of super-saddles, we need another normal forms. Although, for rigid germs, formal normal forms are obtained by Ruggiero, formal conjugacies might diverge. We consider a certain class of rigid germs and show that, for most maps, formal conjugacies diverge. In case they converge, the argument in Inou and the author works.

**Yusuke OKUYAMA**, *Discontinuity of the escape rate of a degenerating meromorphic family of rational maps*

For a holomorphic family  $f_t$  of rational maps of degree  $d > 1$  on the projective line parametrized by the punctured unit disk  $0 < |t| < 1$  whose coefficients extends to meromorphic functions on the whole unit disk  $|t| < 1$ , the function which associates each parameter  $0 < |t| < 1$  with the Lyapunov exponent  $L(f_t)$  of  $f_t$  (with respect to its maximal entropy measure) is continuous and subharmonic on  $0 < |t| < 1$ , and there is a constant  $\eta$  such that the function  $L(f_t) - \eta \log |t|$  has the order  $o(\log |t|^{-1})$  around the puncture  $t = 0$ . In this talk, we provide several examples of families  $f_t$  where the function  $L(f_t) - \eta \log |t|$  on  $0 < |t| < 1$  fails to be bounded around the puncture  $t = 0$ . This is in contrast to the recent result of Favre-Gauthier that the continuity of  $L(f_t) - \eta \log |t|$  on the whole  $|t| < 1$  is always the case for any polynomials family  $f_t$ , and also succeeds in providing a counterexample to a conjecture posed by Favre in 2016. This is a joint work with Professor Laura DeMarco (Northwestern University).

**Hiroki SUMI**, *Weak mean stability in random holomorphic dynamical systems*

We consider random holomorphic dynamical systems generated by holomorphic families of rational maps on the Riemann sphere. We introduce the notion of "weak mean stability" and show several properties of weakly mean stable systems. Also, we show the following (1) (2). (1) Generic random dynamical systems of polynomials of degree two or more are weakly mean stable. (2) Generic random relaxed Newton's method systems are weakly mean stable. For the preprint, see H. Sumi, Negativity of Lyapunov Exponents and Convergence of Generic Random Polynomial Dynamical Systems and Random Relaxed Newton's Methods, <https://arxiv.org/abs/1608.05230>.

**Takato UEHARA**, *On a construction of transcendental K3 surfaces: application of Arnol'd's theorem*

K3 surfaces have recently been intensively investigated from a dynamical point of view, since some of them admit automorphisms with positive topological entropy. In this talk, we construct K3 surfaces by gluing two 9-point blowups of the complex projective plane, where the existence of overlaps is guaranteed by Arnol'd's theorem. A calculation of their period maps shows that such K3 surfaces constitute a large family, including transcendental K3 surfaces. Furthermore, we comment on a relation with automorphisms of K3 surfaces having positive entropy.

**Shigehiro USHIKI**, *Reversible complex dynamical systems and exotic rotation domains*

Complex dynamical systems of 2-dimensional complex manifolds may have exotic rotation domains. Exotic rotation domain is a rotation domain conjugate to the product of annulus and disk. Computer generated pictures strongly suggest the existence of such domains.

## — SHORT TALKS —

**Shunsuke MOROSAWA**, *Dynamics of semigroups of transcendental entire functions*

We consider Dynamics of semigroups of transcendental entire functions. We see some examples of semigroups of transcendental entire functions whose Fatou sets have wandering domains or Baker domains.

**Leticia PARDO SIMÓN**, *Escaping singular orbits in the class B*

In 1989, Eremenko conjectured for transcendental maps that every point in their escaping set can be connected to infinity by a curve in the escaping set. After this was proven to hold for functions in Class B of finite order, the question of when those curves, called "hairs" or "rays", land has been an active topic of research. Even if this might not always be the case, it has been shown for some functions with bounded postsingular set that their Julia set is structured as a (pinched) Cantor Bouquet, that is, an embedding in the plane of a straight brush (with possibly identified endpoints).

In this talk I will consider certain functions with unbounded postsingular set whose singular orbits escape at some minimum speed. In this setting, certain hairs will split when they hit critical points. I will present a new structure for their Julia set as a modified Cantor Bouquet that will allow me to conclude that their hairs, if maybe now with split ends, still land.

**Tomoko SHINOHARA**, *Local invariant set for a rational map of two variables at a fixed indeterminate point*

For some fixed indeterminate point of a rational map of two variables, it is known that there exist a Cantor bouquet or a local stable set. In this talk we discuss properties of a rational mapping to have a local invariant set at a fixed indeterminate point.

**Anand Prakash SINGH**, *On escaping sets of composition of transcendental entire functions*

If  $f$  and  $g$  are transcendental entire functions, then so are  $f \circ g$  and  $g \circ f$ . We discuss here the dynamics of various types of escaping sets and also non-escaping sets of composition of transcendental entire functions  $f \circ g$  and  $g \circ f$  and also its relations with regard to its factors  $f$  and  $g$ . If  $f$  and  $g$  are transcendental entire functions, then so are  $f \circ g$  and  $g \circ f$ . We discuss here the dynamics of various types of escaping sets and also non-escaping sets of composition of transcendental entire functions  $f \circ g$  and  $g \circ f$  and also its relations with regard to its factors  $f$  and  $g$ .

**Kohei UENO**, *Bottcher coordinates for holomorphic skew products*

Let  $f$  be a holomorphic skew product of the form  $f(z, w) = (p(z), q(z, w))$  with a superattracting fixed point at the origin. Under one or two assumptions, we construct a Bottcher coordinate on an invariant open set whose closure contains the fixed point, which conjugates  $f$  to a monomial map. The monomial map and the open set are determined by the order of  $p$  at the origin and the Newton polygon of  $q$ .